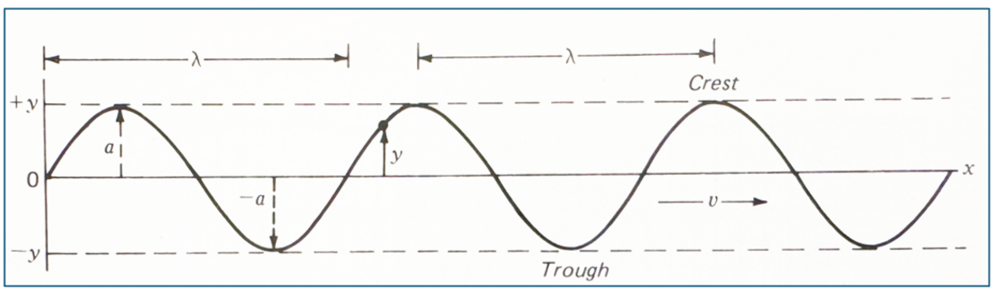
Making Sense of Interference

Wave Optics

Characteristics of Wave Motion
Light has characteristics consistent with both a wave and a particle nature.
A wave is a disturbance that varies in both space and time.
Harmonic waves are often used to represent light waves.

Types of Waves
Longitudinal: oscillations in the same direction as light travels
Transverse: oscillations perpendicular to the direction of light
Sinusoidal: use sine function to plot oscillation
Harmonic: wave repeats itself, periodical and symmetrical displacement



(Jenkins & White 1976)

Wavelength (λ): distance between consecutive corresponding points of the same phase such as two zero crossings as illustrated above.
Amplitude (α): maximum displacement of the wave along the y axis.
Frequency (f): number of cycles per unit of time. Usually expressed in Hertz.

Important Relationships

Velocity of Light (ν)	Frequency (f)	Angular Frequency (ω)
$v = f\lambda$	$f = \frac{1}{T}$	$\omega = fk$ $k = \frac{2\pi}{\lambda}$

Cycles

One complete cycle (λ) = 2π radians (360°)

½ $\lambda = \pi$ radians (180°)

¼ $\lambda = \pi/2$ radians (90°)

Light can be described as either a sine wave or cosine wave.
The difference between sine and cosine is a $\pi/2$ (90 degree) shift.

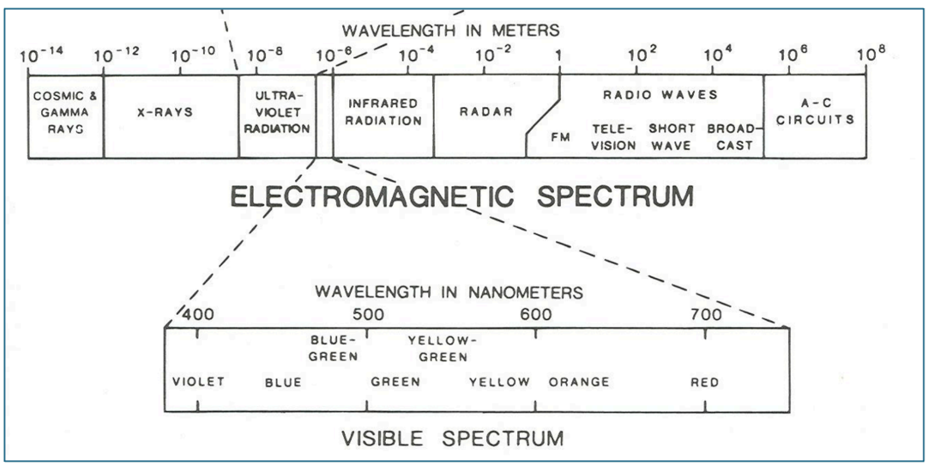
Classification of the electromagnetic spectrum

Visible light is an electromagnetic (EM) wave.

EM waves are characterized by oscillations in the electric field (E) and magnetic field (B) which are perpendicular (90 degrees apart).

The Electromagnetic Spectrum (illustrated below) is the range of EM radiation arranged from shortest to longest.

The range of visible light is approximately 380 to 760nm (10^{-9} meters).



Total and partial coherence

Coherence of light refers to a situation in which two waves have a constant relative phase.
Coherence is the degree to which peaks and troughs of wave are matched.
The amount of coherence can be measured by examining the size of the interference fringes.
The visibility or contrast of an inference pattern is controlled by coherence.
A high degree of coherence means the waves have the same frequency, wavelength, amplitude and phase.
The higher the variation in frequency / wavelength, the lower the coherence.
Lasers are highly coherent while common light sources are highly incoherent.

Interference

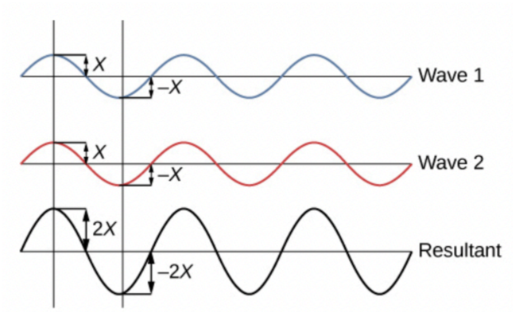
Superposition of Waves

If two sine waves with the same frequency and wavelength are combined, the resulting wave is also sinusoidal with the same frequency and wavelength.
In Phase: two waves of the same frequency for which one crest superposes on the other crest.
Out of Phase (antiphase): when a wave's crest superposes on another wave's trough.

Optical Path Length: nx where n is index of material and x is distance from source.
Optical Path Difference: $n(x_2 - x_1)$ where x_1 and x2 are distances from sources for two harmonic waves.

Constructive Interference

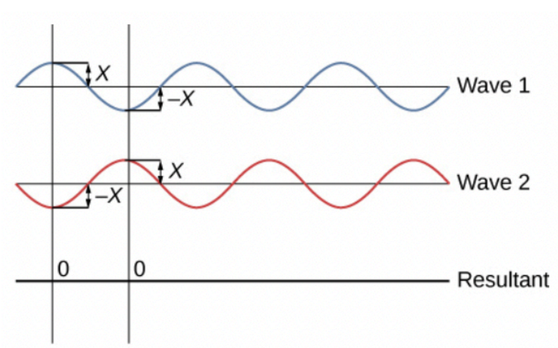
Occurs when the crest of one wave falls upon the crest of another wave, or very nearly so.
Waves have the same frequency.



Credit: UCF Physics 3 (Ling, Sanny, & Moebs)

Destructive Interference

Occurs when the crest of one wave falls on the crest of another.
Waves have the same frequency.



Credit: UCF Physics 3 (Ling, Sanny, & Moebs)

Interference Patterns

To observe interference patterns, we must have coherent light.

Interference Pattern

Credit: UCF Physics 3 (Ling, Sanny, & Mo

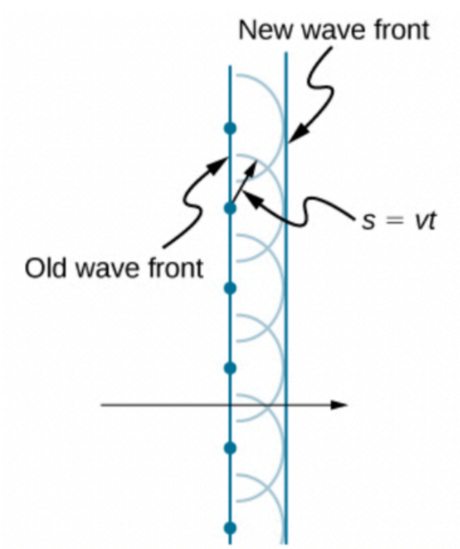
Huygen’s Principle

To understand diffraction and interference you must understand Huygen’s Principle that says every wavefront is made up of an infinite number of wavelets propagating from an infinite number of points along the previous wavefront.

Each wavelet has the same wavelength.

This explains why light will change directions when it encounters an obstacle.

Huygens principle



Credit: UCF Physics 3 (Ling, Sanny, & Moebs)

Intensity of Maxima and Minima

$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\ cos\delta$

(δ is the phase difference between 2 waves)

Intensity at Maxima (I_{max})

I_{max} occurs when cosδ = 1

This occurs when $\delta = 0, \pm 2\pi, \pm 4\pi, \ldots$

This is when optical path difference is an integer number of wavelengths.

Optical Path Difference (OPD) = mλ, where m = 0, ±1, ±2, ±3, ...

In this case the phase difference between the two wave is an integer multiple of 2π and disturbance are
in phase.

Crest of one wave superposed on crest of another and complete constructive interference occurs.

Young's Double Slit Experiment

To demonstrate the wave nature of light Young devised an experiment using coherent light passing through two narrow slit apertures and forming and image on a screen.

The image formed on the screen is called an interference pattern and is produced as the waves from the two light sources (created by the apertures) interfere with each other.

The interference pattern created on the screen consists of alternating bright and dark fringes.

I_{max} (maximum intensity) corresponds to the center of the bright fringe.

I_{min} (minimum intensity) corresponds to the center of the dark fringe.

$\Delta y = s\lambda / na$

Where: Δy: the distance between fringes (dark to dark or light to light)

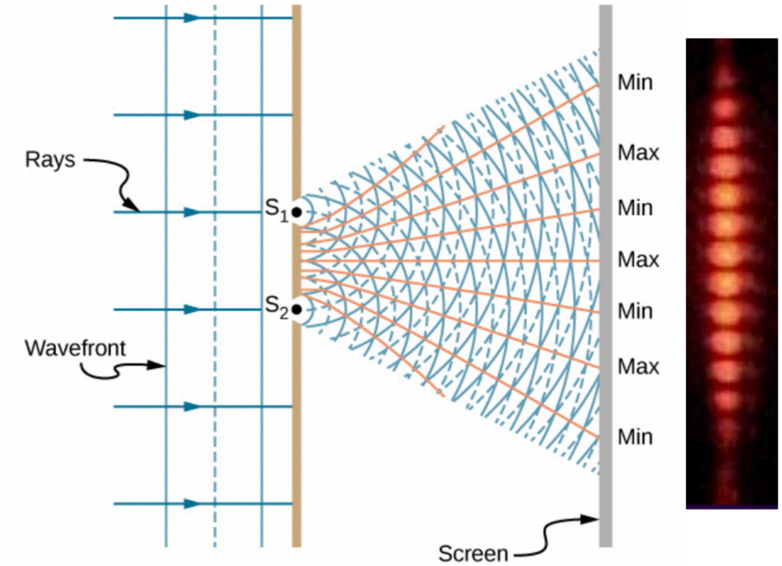
s: distance from slit to screen

λ: wavelength of light in air

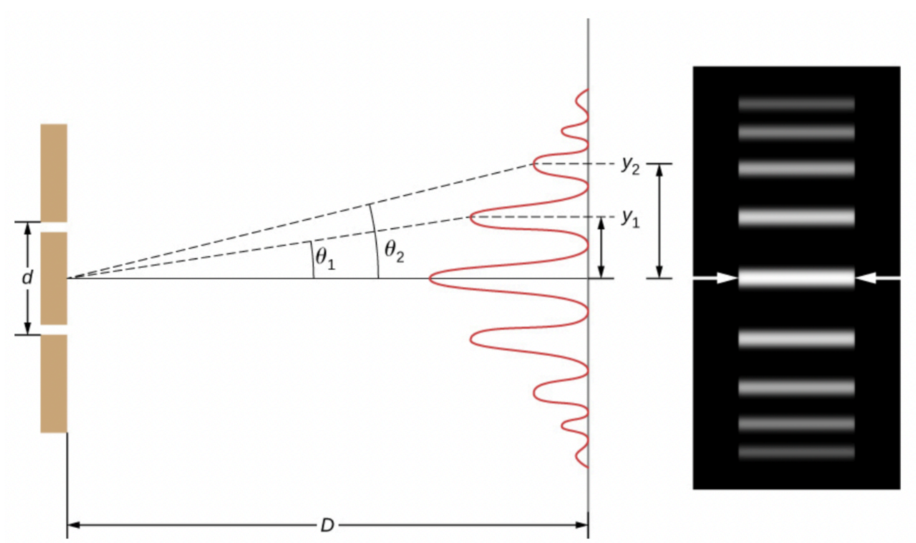
n: index of medium (usually air)

a: distance between slits

Interference from Double Slit



Credit: UCF Physics 3 (Ling, Sanny, & Moebs)



Credit: UCF Physics 3 (Ling, Sanny, & Moebs)

A larger Δy means the pattern is more spread apart.
As s or λ get larger, Δy gets larger
As n or a gets larger, Δy gets small

Interference from Thin Films

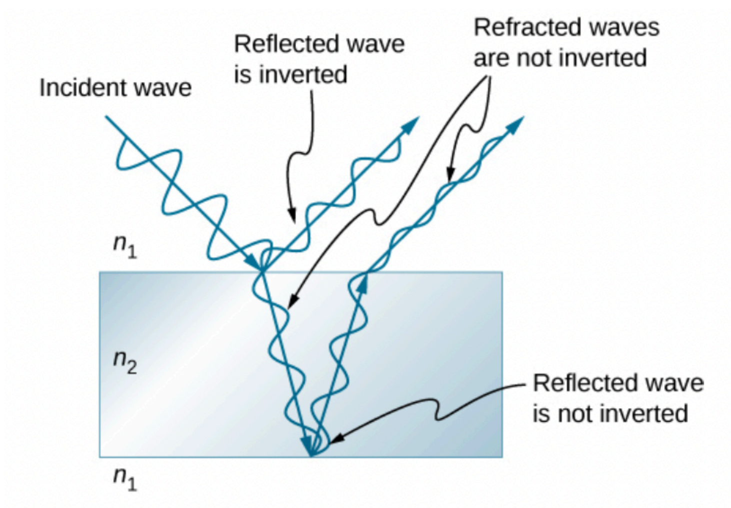
A thin film may be a soap film, an oil slick, or an air film between two glass plates.
For example, a soap bubble can be considered a thin film of water surrounded by air.
The bright colors seen in an oil slick or in a soap bubble are the result of interference.
When incident light reflects off the front and back surfaces of a thin film the two waves created can interfere with each other forming an interference pattern.

Depending on the wavelength of light and the thickness of the film the reflected light can constructively or destructively interfere.
Varying thickness across a thin film creates different colors of reflection.
Thin films appear darkest where they are the thinnest.
As the film becomes thicker the separation of the reflected rays increases.
If white light is incident, colored fringes are seen.

For a soap bubble or oil film only the first reflection has a π phase shift so if a film has an optical thickness of $\lambda/4$ the reflected waves will constructively interfere.

Reducing the angle of incidence will reduce the separation of the rays and at incidence angles near normal the interference pattern will be easiest to observe.

Reflections from a Thin Film

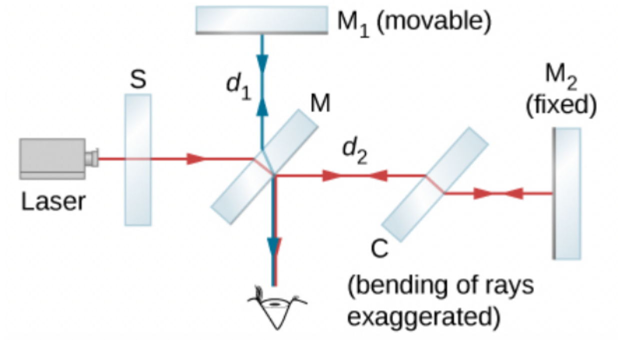


Credit: UCF Physics 3 (Ling, Sanny, & Moebs)

Light traveling from a lower index medium to a higher index medium undergoes a π phase shift.
For a soap bubble (water surrounded by air) reflection at the front surface results in a π phase change but reflection from the back surface does not.
Dark areas correspond to areas with destructive interference.

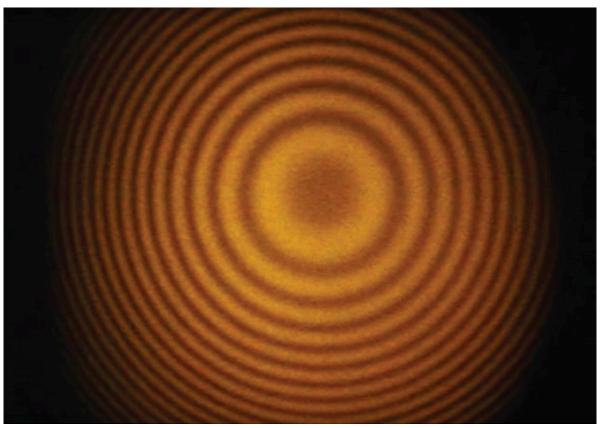
Michelson Interferometer

The Michelson interferometer works by producing interference fringes by splitting a light beam into two parts and then recombing them after they have traveled different optical paths. A beam of monochromatic light is incident on a half silvered mirror that reflects half of the beam and allows the other half to pass through it.



(credit UCF Physics)

The two beams are reflected so that they can be observed at M on the diagram above. They two beam are coherent so the result is an interference pattern (see next image).



(credit UCF Physics)

The path difference is $2d_1 - 2d_2$ where d_1 is the distance between M and M_1 and d_2 is the difference between M an M_2.

By counting the number of fringes (m) passing a given point as M1 is moved, very small displacements can be measured. Each time the pattern shifts by one full ring, the mirror has been moved by ½ wavelength.

$$d = m\frac{\lambda_0}{2}$$

Newton's Rings

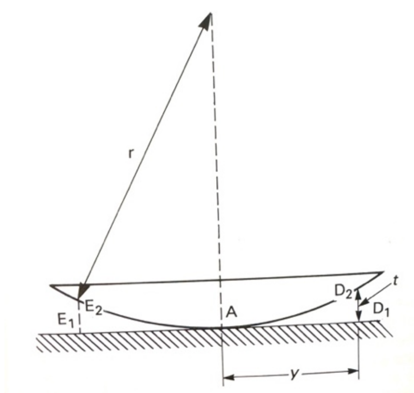
When a convex lens is placed against a flat glass surface and an air wedge is formed between them an interference pattern known as Newton's Rings can be seen.
When monochromatic light is incident, alternating bright and dark rings can be seen.
If the incident light is white, then colored rings will form.
The thickness of the air wedge (film) at any given point will determine whether the interference will be constructive or destructive.

The radius of the convex surface can be determined by: $R = r_m^2(m\lambda)$ where R is the radius.

r_m^2: the radius from center of pattern to m^{th} dark ring

m: the ring number chosen to be measured (1, 2, 3, ...)

λ: wavelength of reflected light



(credit: Freeman 1990)

Antireflective (AR) Coatings

AR coatings provide a practical example of interference.
Light incident on a lens surface can be reflected, absorbed, or transmitted. The reflection of light incident normal to an optical lens surface will be reflected according to the following formula:

$$I_R = \frac{(n'-n)^2}{(n'+n)^2}(I)$$

I_R is the percentage of the incident light that is reflected by the lens surface.
I is the light incident on the surface (for front surface this is 1.00 or 100%).

For a lens with an index of 1.50:

Reflection at front surface: 4.0%

Reflection at back surface: 3.8%

Total reflection of uncoated lens: 7.8%

Reflections occur at both lens surfaces (the interface of air and the front surface and the interface of the back surface of the lens and air):

Spectacle Lens Reflection

When light reflects from a low-high index interface it undergoes a π phase shift (π = 180° or $\lambda/2$).
This means that peaks and troughs switch places.
There is no phase shift when light is transmitted or when it reflects off a high-low interface.

A thin antireflective coating can be applied to a lens surface that will reduce the intensity of reflections using the principle of interference.
Reflected light destructively interferes while the transmitted light constructively interferes.
AR coatings increase the transmission through a lens and decrease the intensity of the refected light.

Index of the ideal AR Coating

A simple formula can be used to find the required index of refraction for the ideal AR coating for any lens given the index of the lens:

$$n_{coating} = \sqrt{n_{lens}}$$

Thickness of the ideal AR Coating

The thickness of an ideal AR coating can be expressed in two ways:

$$Ideal\ Optical\ Thickness = \lambda / 4$$

$$Ideal\ Physical\ Thickness = \lambda / (4n_c)$$

Light travels a distance of ¼ λ through a coating and another ¼ λ back through the coating after reflection which is the same as a π phase shift or a 180° phase shift.
AR coatings work by producing two reflections which interfere destructively with each other.

Problem Types

Changes in width of fringes in double slit experiment
Calculate the index and thickness of an ideal AR coating
Calculations with interferometer measurements
Thin film thickness / color
Newton' rings: radius / power of surface

Problem 1: find wavelength of Laser source given separation of slits, distance to screen, and distance between fringes:

What is the wavelength of a light source (laser) used with a double slit if the slits are separated by 0.085mm and the pattern on a screen 2.3m away has a separation of fringes of 1.2cm:

Our relevant formula is: $\mathbf{\Delta y = s\lambda / na}$

Where: Δy: the distance between fringes (dark to dark or light to light)

s: distance from slit to screen

λ: wavelength of light in air

n: index of medium (usually air)

a: distance between slits

We want to find wavelength of source (λ).

We are given Δy, s, and a. Assume n = 1.00 since not given

$$\lambda = \frac{\Delta y n a}{s}$$

λ = (12mm)(1)(0.085mm) / 2300mm

λ = 1.02mm / 2300mm

λ = 0.000443mm

λ = 443nm

Problem 2: what is the effect on the separation of bright fringes in the previous problem if the distance to the screen is reduced to half the original distance:

$\mathbf{\Delta y = s\lambda / na}$

First, what is the effect of reducing screen distance?

Screen distance (s) is in the numerator, therefore, reducing its magnitude will result in a decrease in **Δy.** So, the separation of bright fringes should decrease.

$\mathbf{\Delta y = s\lambda / na}$

Δy = (1150mm)(0.000443mm) / 0.085mm

Δy = 0.509 / 0.085

Δy = 5.99mm or 0.60cm

Problem 3: describe how changing each factor in the double slit experiment effect the separation of bright fringes in the interference pattern:

$\Delta y = s\lambda / na$

Screen Distance (s): decreasing screen distance decreases Δy
Wavelength of Source (λ): decreasing wavelength decrease Δy
Increasing Index of Medium (n): increasing index deceases Δy
Slit Separation (a): Increasing Slit Separation decreases Δy

Problem 4: find the index and optical thickness of the ideal Anti Reflective coating to be used on a high index plastic spectacle lens with an index of refraction of 1.65 if the lens is to maximally transmit light in the center of the visible light spectrum (555nm):

The index of the coating (which is ALWAYS lower than the lens) is the square root of the index of the lens.

In this case the index of the coating is:

$\sqrt{1.65}$ = 1.28

Index of AR coating = 1.28

The optical thickness is calculated using the wavelength for which you want to maximize transmission (reduce reflections) for the lens:

Optical thickness = λ / 4
Optical thickness for 555nm = 555 /4
Optical thickness of AR coating = 139nm

Problem 5: Interferometry
Circular fringes are observed with a Michelson interferometer when illuminated by monochromatic light. When the mirror is moved 0.073mm, 300 fringes pass a point near the center of the pattern. What is the wavelength of incident light:

We can use the relationship:

$$d = m\frac{\lambda_0}{2}$$

We are solving for λ
λ = 2(0.073mm) / 300
λ = 0.000487mm
λ = 487nm

Problem 6: Thin films
Light of wavelength 489nm is incident on a soap film (n = 1.33) in air. What is the smallest thickness that gives a maximum in the reflected light:

This is a physical thickness problem where $t = \lambda/4n$.

$t = 489nm/4(1.33)$
$t = 489/5.32$
$t = 92nm$

Problem 7: Newton's Rings
A lens (n = 1.50) is illuminated with light with a wavelength of 546nm. If the diameter of the 10^{th} dark ring is 2.10mm, what is the radius and power of the convex surface:

Find the radius of the surface.
We can use the following relationship:

$R = r^2_m / (m\lambda)$
R is the radius of the surface and r^2_m is radius of the m^{th} dark ring

$R = (1.05mm)^2 / 10(0.000546mm)$
$R = 1.10 / 0.00546$
$R = 201.92mm$
$R = 20.2cm$ or $0.202m$

Find the power of the surface.
$F = (n'-n)/r$
$F = 0.50 / 0.202$
$F = +2.48D$